

# **ANALYTICAL METHODS FOR SIMULATION OF HARD PROJECTILE IMPACT ON REINFORCED CONCRETE STRUCTURES**

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## **ABSTRACT**

In this paper, a simplified and robust mechanical analysis approach for predicting the load-bearing capacity of reinforced concrete structures under hard impact loads is presented. The mechanical principles of the enhanced engineering model are based on a nonlinear two degree of freedom (TDOF) system by Schlüter [1], which was extended for applications on hard impact scenarios. For verification and validation purposes and to ensure the physical correctness of the failure mechanism, experimental data from the Technical Research Centre of Finland (VTT) [2] are used.

## **INTRODUCTION**

Reinforced concrete structures such as nuclear power plants, dams as well as military installations and their weapons depots require a very high standard of safety. Due to the catastrophic consequences of damage or destruction of these structures, adequate protection against impact loads must be ensured through the development of appropriate simulation and design models. In the worst-case scenario of an aircraft crash, local and global damage is caused to the structure. For a better understanding of the damage mechanisms, experimental tests as well as analytical and numerical models can be investigated. Based thereon the damage can be quantified and design guidelines developed.

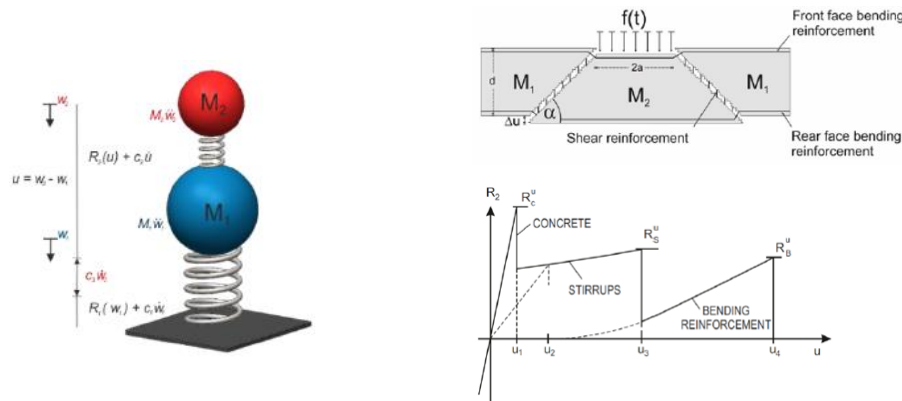
Basically, impact scenarios can be divided into hard and soft impacts. In case of a soft impact, the projectile is deformable relative to the target structure and the load-time function can be determined using simplified methods, e. g. by Riera [3]. In the event of a hard impact, the calculation of capacity and damage effects is very sophisticated as the contact actions and target reactions are strongly coupled. Therefore, the investigation of hard missile impacts requires several adjustments to existing analytical models.

## **CEB MODEL ACCORDING TO SCHLÜTER**

The CEB (Comité Euro-International du Béton) model according to Schlüter [1] and CEB [4] represents a simplified analytical model for estimating the system response of a reinforced concrete slab under missile impact. In combination with a suitable load approach, the analytical model describes all relevant mechanisms in a physically adequate manner. In the CEB model, the reinforced concrete slab is idealised as a system with two degrees of freedom with the following equations of motion (see equations (1) and (2) and figure 1).

$$M_1 \cdot \ddot{w}_1 + c_1 \cdot \dot{w}_1 + R_1(w_1) - R_2(u) - c_2 \cdot \dot{u} = 0 \quad (1)$$

$$M_2 \cdot \ddot{w}_2 + c_2 \cdot \dot{u} + c_3 \cdot \dot{w}_2 + R_2(u) - F(t) = 0 \quad (2)$$

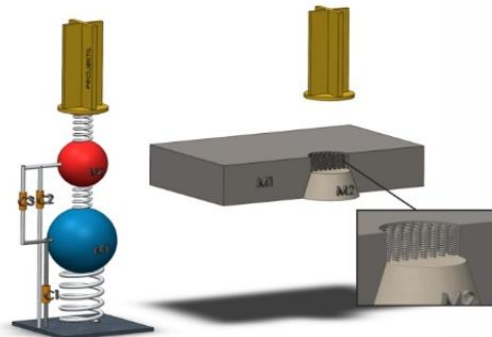


**Figure 1:** CEB model according to Schlüter [5]

The two masses  $M_1$  and  $M_2$  are coupled with nonlinear springs and dampers (see figure 1). In the analytical model,  $M_1$  and  $R_1(w_1)$  represent the global deformation characteristics in bending.  $M_2$  and  $R_2(u)$  represent the local behaviour of the punching cone in the area of impact relative to the rest of the plate ( $u = w_2 - w_1$ ). Damping is also included in the CEB model to represent the amount of energy dissipation from internal damage of the concrete and the dowel action of the bending reinforcement. Before impact, the assumed punching cone  $M_2$  is monolithically connected to the remaining concrete slab. The resistance  $R_1$  is idealized as an elastoplastic spring describing bending of the plate. The separation of punching cone and slab occurs when the concrete tensile strength is exceeded. The local resistance  $R_2(u)$  consists of three components, which can be idealized as three parallel connected springs, describing the contribution of the concrete, the stirrups and the bending reinforcement. Figure 1 clarifies the three components of the local resistance  $R_2(u)$ .

## ADJUSTMENTS FOR HARD MISSILE IMPACT

For the investigation of hard missile impact, the interaction between the target and the impacting projectile as well as the process of penetration of the projectile need to be considered (see figure 2).



**Figure 2:** Extended model for hard missile impact [5]

Based on the CEB model according to Schlüter which only works for soft missile impact, adjustments are presented in the following that allow the consideration of hard missile impact.

## LOAD INTERACTION

To determine the target response to a hard missile impact, it is desirable to know the impact force-time history or at least the duration of impact. Using the “Theory of Penetration” based on Kennedy [6] the time-history of the impact force can be approximated by equation (4). The equation of motion for the impacting projectile at any time  $t_i$  after the impact is given by equation (3).

$$M_p \cdot \frac{d^2 x_i}{dt_i^2} = M_p \cdot \frac{v_i dv_i}{dx_i} = -P_i \cdot A_c = F_i \quad (3)$$

$$P_i = \frac{263820}{KN} \cdot \left( \frac{v_i}{12000d} \right)^{0,2} \cdot g_{z_i} \quad (4)$$

$$g_{z_i} = \begin{cases} \left( \frac{x_i}{2d} \right) & \text{für } \frac{x_i}{d} \leq 2,0 \\ 1,0 & \text{für } \frac{x_i}{d} \geq 2,0 \end{cases} \quad (5)$$

Where  $M_p$  is the mass of the projectile,  $v_i = \frac{dx_i}{dt_i}$ ,  $d$  is the slab thickness and  $F_i$  represents the total impact force at the timestep  $t_i$  multiplied with the contact area of the projectile  $A_c$ .  $N$  is a nose shape factor for the missile. For a flat nosed missile  $N$  is equal to 0,72, for blunt nosed missile it is 0,82, for average missile nose with spherical end 1,00 and 1,14 for a very sharp nose shape.  $K$  is a concrete penetrability factor that is a function of the concrete strength [6]. The formula is derived for American units and must be converted consistently if necessary.

In case of a non-deformable missile impact, the impact process should be considered in two different phases. In the first phase, the projectile penetrates the reinforced concrete structure and creates an almost cylindrical penetration form as long as the concrete spring  $R_c^u$  is active. As a result of the penetration due to the projectile the total slab thickness  $d$  decreases at each time step  $t_i$ . To consider this adjustment for hard missile impacts, the load-bearing capacity of the concrete  $R_c^u$  is modified from the original CEB model in equation (6) where  $a$  is the projectile radius and  $f_{ct}$  represents the tensile strength of concrete.

$$R_c^u = \left[ \left( a + \frac{d-x_i}{\tan(\alpha)} \right)^2 - a^2 \right] \pi \cdot f_{ct} \quad (6)$$

In addition, the effective height of the plate  $h_{eff}$  is also reduced resulting in a decrease in fracture deformation of  $u_1$  (see equations (7) and (8)).

$$h_{eff} = 0,5 \cdot (d - x_i) \quad (7)$$

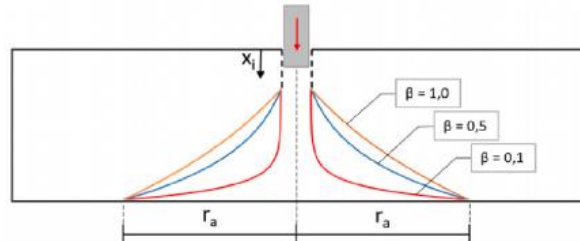
$$u_1 = h_{eff} \cdot \frac{2}{3} \cdot \frac{f_{ct}}{E_c} \quad (8)$$

## MASS OF THE PUNCHING CONE

Schlüter simplifies the calculation of the mass of the punching cone  $M_2$  on the assumption of a linear cone shape. Experimental investigations concerning the punching cone in Just et al. [7] and NEA [8] have shown a dependence between the shape of the punching cone and the function of projectile speed as well as plate thickness  $d$ . To improve the calculation of  $M_2$ , the shape of the cone will be

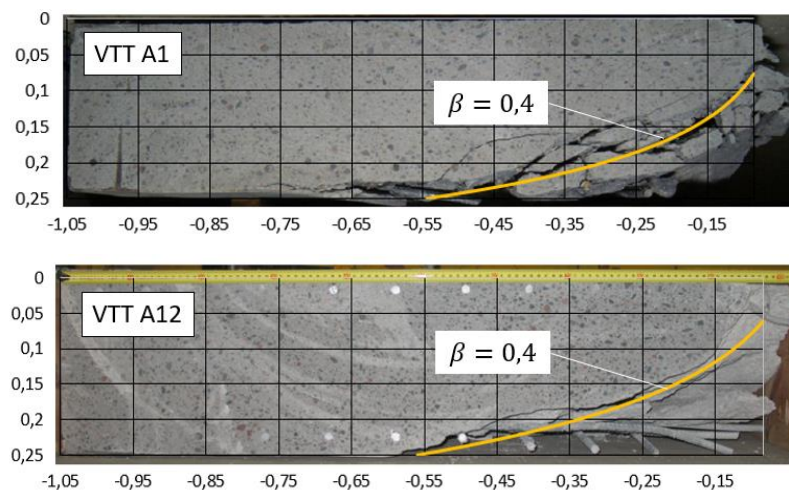
estimated with an exponential shape function based on an input parameter  $\beta$  shown in equation (9) and visualized in figure 3.

$$s_f(d_i) = d \cdot \left( c \cdot e^{\left(\frac{1}{\beta}\right) \left(\frac{x_i}{d}\right)} - b \right) \quad (9)$$



**Figure 3:** Shear fracture zone

In equation (9),  $d$  is the slab thickness and  $r_a$  is the assumed punching cone radius. The parameters  $c$  and  $b$  are fixed values that are defined by the boundary values of the function,  $s_f(x_i) = 0,5 \cdot D$  and  $s_f(d) = r_a$ . Figure 4 presents the application of the adjusted approach for the VTT tests A1 and A12.



**Figure 4:** Cross-section of the RC structure as well as the assumed punching cone shape

The experimental documentation of the VTT tests IRIS P2, IRIS P3 as well as P3 and P12 included an estimation of the mass  $M_2$ . If these are calculated according to the exponential method, the calculated and experimentally determined masses  $M_2$  are in good agreement. Table 1 shows that the exponential approach leads to a significant result improvement compared to the linear calculation.

**Table 1:** Comparison of the calculated and experimentally determined masses  $M_2$

Test	$M_2$ linear [kg]	$M_2$ exponential [kg]	$M_2$ experimental [kg]
IRIS P2	172	118	116
IRIS P3	154	108	121
VTT P3	240	125	125
VTT P12	307	160	160

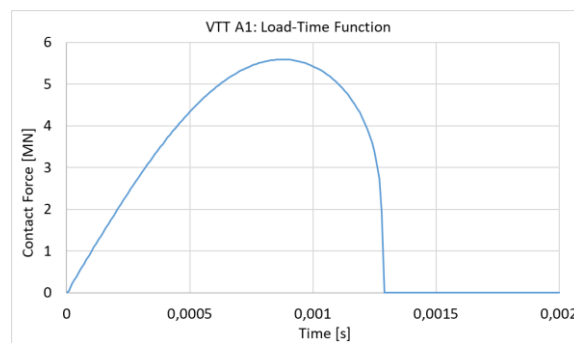
## STUDIES IN SELECTED IMPACT TESTS

Table 2 shows the test data of the selected impact test conducted by the Technical Research Centre of Finland VTT [9].

**Table 2:** Test data of impact test VTT A1

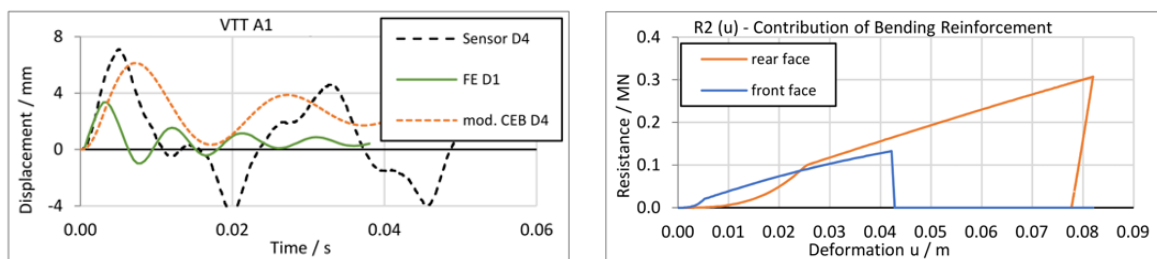
Test	$M_p$ [kg]	$v_p$ [m/s]	$D$ [m]	Dimensions [m]	$f_c$ [N/mm <sup>2</sup> ]	Reinforcement [%]
VTT A1	47,5	101	0,1683	2,10 x 2,10 x 0,25	60	0,35

Experimental data of the test VTT A1 is used for verification purposes of the simplified mechanical analytical model. Initially, the load-time function is calculated according to Kennedy [6] (see figure 5).



**Figure 5:** Load-time-function of the test VTT A1 according to Kennedy [6]

This load approach is applied to the extended nonlinear two degree of freedom system. Figure 6 shows the target response to the hard impact. The ultimate load capacity of the reinforced concrete slab was not exceeded as the local spring  $R_2$  seems to be intact in the analytical model. The displacement-time history illustrated in figure 6 (left) shows that the global damage is slightly overestimated. The overall target response of the adjusted CEB model is in good agreement with the experimental data as the damage mode is predicted correctly.



**Figure 6:** Calculated and measured deformation of VTT A1

## CONCLUSION

In this paper, an analytical model for simulation of soft missile impact on reinforcement concrete structures is introduced. Furthermore, adjustments for this model for application on hard missile impact are presented and validated with experimental test data of the Technical Research Centre VTT of Finland.

For verification purposes of the adjusted analytical model, the results of the hard missile test VTT A1 are used. The comparison between the analytical model and the measured data shows good agreements. The essential mechanical behaviour of the reinforced concrete slab can be reproduced by the adjusted CEB model. The evaluation of the results must consider that particular input parameters such as the punching angle and damping are assumed. In order to increase the robustness of the analytically calculated results, the adjusted CEB model should be tested on a larger experimental base.

## ACKNOWLEDGEMENT

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